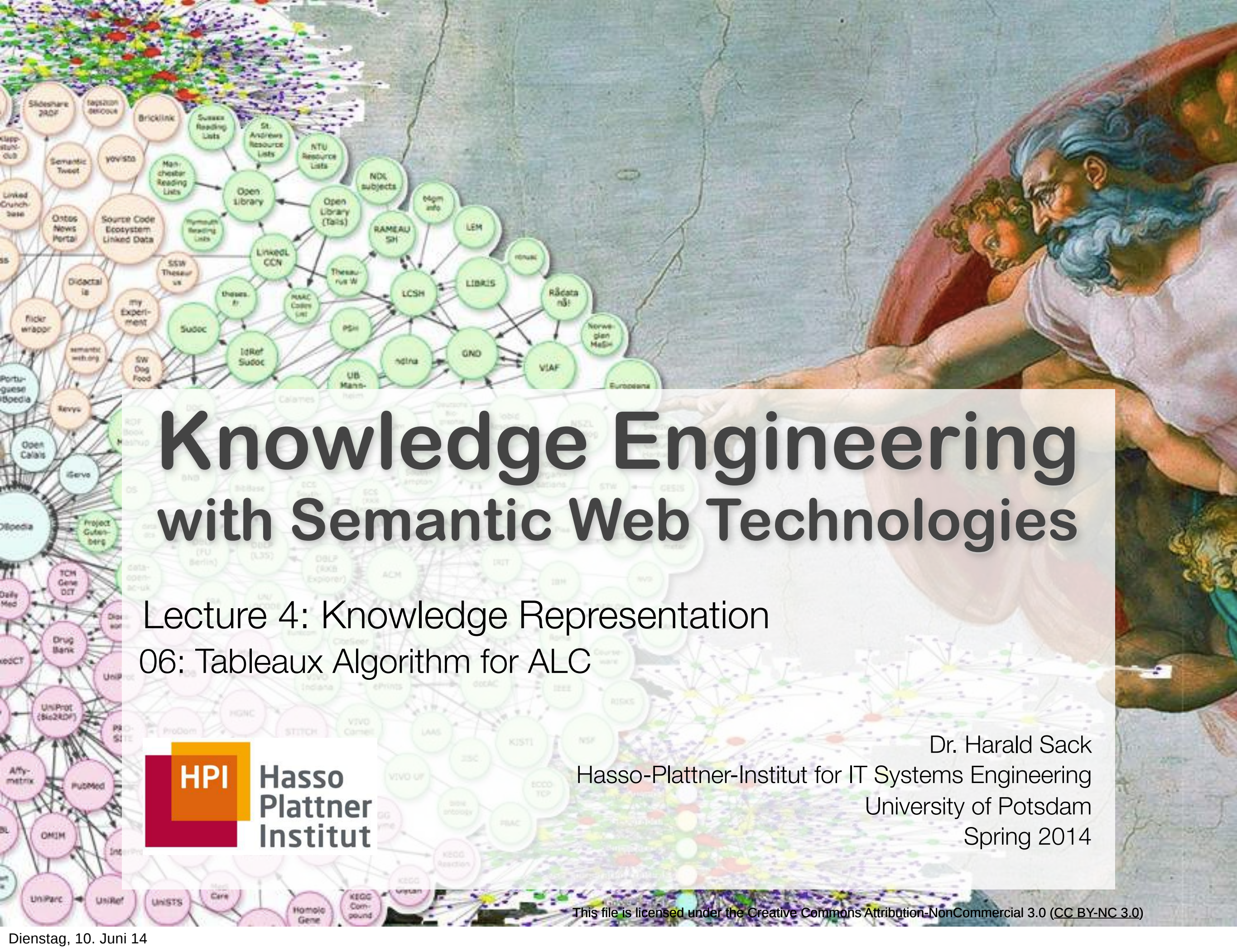




# Knowledge Engineering with Semantic Web Technologies

Lecture 4: Knowledge Representation  
06: Tableaux Algorithm for ALC



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# **Lecture 4: Knowledge Representations I**

Open HPI Course: Knowledge Engineering with Semantic Web Technologies

Handwritten text in German, likely a list of names or a table, written in a cursive script. The text is arranged in four rows. The first row contains 8 entries, the second row contains 8 entries, the third row contains 8 entries, and the fourth row contains 8 entries. The handwriting is somewhat messy and includes many corrections and deletions.

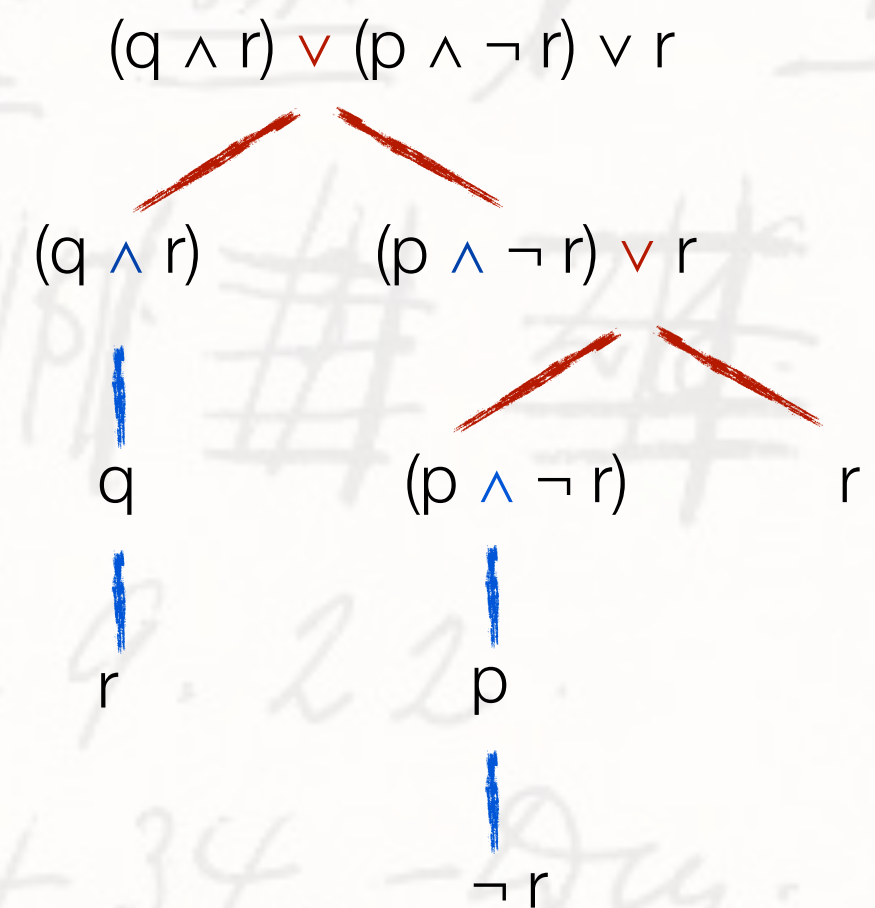
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## 07 Tableaux Algorithm for ALC

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Lecture 04: Knowledge Representations Pt.1

# Short Recapitulation

- Proof algorithm to check the consistency of a logical formula by inferring that its negation is a contradiction (**proof by refutation**).
- Construct **Tree**, where each **node** is marked with a logical formula. A **path** from the root to a leaf is the **conjunction** of all formulas represented within the nodes of the path; a **branch** of the path represents a **disjunction**.
- The tree is created by successive application of the **Tableaux Extension Rules**.



# Tableaux Algorithm for Description Logics

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- Transformation to **Negation normalform NNF** necessary
  - Let  $W$  be a knowledge base,
    - Substitute  $C \equiv D$  by  $C \sqsubseteq D$  and  $D \sqsubseteq C$
    - Substitute  $C \sqsubseteq D$  by  $\neg C \sqcup D$ .
  - Apply the NNF Transformations from the next page
  - Resulting knowledge base  $NNF(W)$ 
    - Negation Normalform of  $W$ .
    - Negation is placed directly in front of atomic classes.

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- NNF Transformations

$\text{NNF}(C) = C$ , if  $C$  is atomic  
 $\text{NNF}(\neg C) = \neg C$ , if  $C$  is atomic  
 $\text{NNF}(\neg\neg C) = \text{NNF}(C)$   
 $\text{NNF}(C \sqcup D) = \text{NNF}(C) \sqcup \text{NNF}(D)$   
 $\text{NNF}(C \sqcap D) = \text{NNF}(C) \sqcap \text{NNF}(D)$   
 $\text{NNF}(\neg(C \sqcup D)) = \text{NNF}(\neg C) \sqcap \text{NNF}(\neg D)$   
 $\text{NNF}(\neg(C \sqcap D)) = \text{NNF}(\neg C) \sqcup \text{NNF}(\neg D)$   
 $\text{NNF}(\forall R.C) = \forall R.\text{NNF}(C)$   
 $\text{NNF}(\exists R.C) = \exists R.\text{NNF}(C)$   
 $\text{NNF}(\neg\forall R.C) = \exists R.\text{NNF}(\neg C)$   
 $\text{NNF}(\neg\exists R.C) = \forall R.\text{NNF}(\neg C)$

- $W$  and  $\text{NNF}(W)$  are logically equivalent.

# Tableaux Transformation in Negation Normalform

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- Example:  $P \sqsubseteq (E \sqcap U) \sqcup \neg(\neg E \sqcup D)$

- In NNF:  $\neg P \sqcup (E \sqcap U) \sqcup (E \sqcap \neg D)$

$$C \sqsubseteq D = \neg C \sqcup D$$

$$\neg(C \sqcup D) = \neg C \sqcap \neg D$$

# Tableaux Extension Rules for DL

| Selection                      | Action                                                   |
|--------------------------------|----------------------------------------------------------|
| $C(a) \in W \text{ (ABox)}$    | Add $C(a)$                                               |
| $R(a, b) \in W \text{ (ABox)}$ | Add $R(a, b)$                                            |
| $C \in W \text{ (TBox)}$       | Add $C(a)$ for a <b>known Individual</b> $a$             |
| $(C \sqcap D)(a) \in A$        | Add $C(a)$ and $D(a)$                                    |
| $(C \sqcup D)(a) \in A$        | Split the path. Add (1) $C(a)$ and (2) $D(a)$            |
| $(\exists R.C)(a) \in A$       | Add $R(a, b)$ and $C(b)$ for a <b>new Individual</b> $b$ |
| $(\forall R.C)(a) \in A$       | if $R(a, b) \in A$ , then add $C(b)$                     |

- If the resulting tableaux is closed, the original knowledge base is unsatisfiable.
- Only select elements that lead to new elements within the tableaux. If this is not possible, then the algorithm terminates and the original knowledge base is satisfiable.

# Tableaux Algorithm (DL) - Example

- P ... Professor
- E ... Person
- U ... University Employee
- D ... Student
- Knowledge Base:  $P \sqsubseteq (E \sqcap U) \sqcup (E \sqcap \neg D)$
- We want to proof: Is  $P \sqsubseteq E$  a logical consequence?
- Knowledge Base (with [negated] query) in NNF:  
 $\{\neg P \sqcup (E \sqcap U) \sqcup (E \sqcap \neg D), (P \sqcap \neg E)(a)\}$

$$C \sqsubseteq D = \neg C \sqcup D$$

$$\neg(C \sqcup D) = \neg C \sqcap \neg D$$

# Tableaux Algorithm (DL) - Example

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- Knowledge Base  $W: \{\neg P \sqcup (E \sqcap U) \sqcup (E \sqcap \neg D), (P \sqcap \neg E)(a)\}$
- Tableaux  $A$ :

(1)  $(P \sqcap \neg E)(a)$  (from knowledge base)

(2| $\alpha$  from 1)  $P(a)$

(3| $\alpha$  from 1)  $\neg E(a)$

(4)  $(\neg P \sqcup (E \sqcap U) \sqcup (E \sqcap \neg D))(a)$  (from knowledge base)

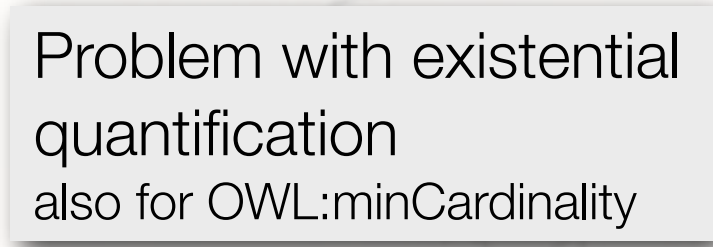
(5| $\beta$  from 4)  $\neg P(a)$  | (6)  $((E \sqcap U) \sqcup (E \sqcap \neg D))(a)$

(7| $\beta$  from 6)  $(E \sqcap U)(a)$  | (8)  $(E \sqcap \neg D)(a)$

(9| $\alpha$  from 7)  $E(a)$  (10| $\alpha$  from 8)  $E(a)$

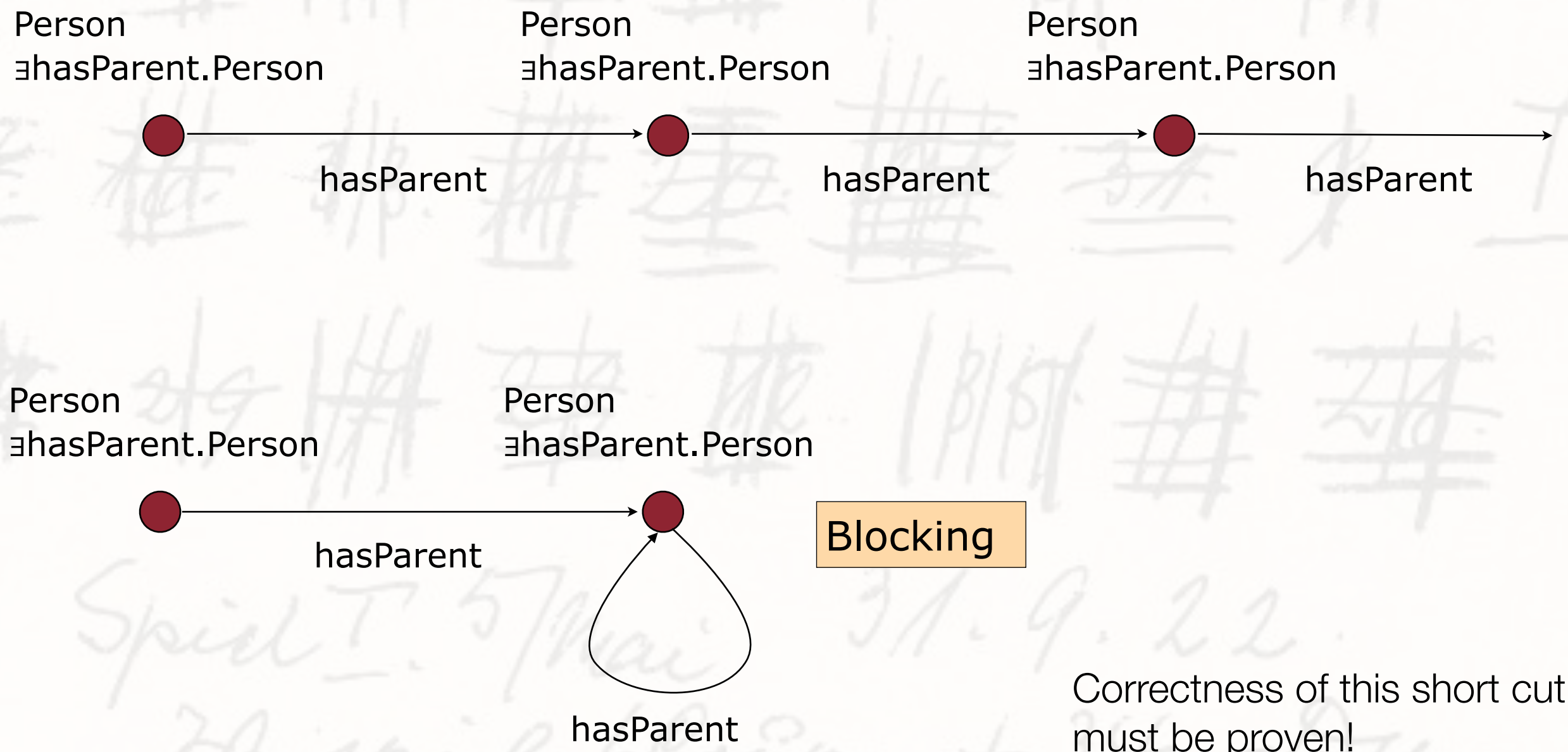
(11| $\alpha$  from 7)  $U(a)$  (12| $\alpha$  from 8)  $\neg D(a)$

- 11



# Idea of Blocking

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# Tableaux Algorithm (DL) with Blocking

- Knowledge Base:  $\neg \text{Person} \sqcup \exists \text{hasParent}.\text{Person}$
- infer:  $\neg \text{Person}(\text{Bill})$

Person(Bill)

$(\neg \text{Person} \sqcup \exists \text{hasParent}.\text{Person})(\text{Bill})$

$\neg \text{Person}(\text{Bill})$

$\exists \text{hasParent}.\text{Person}(\text{Bill})$

hasParent(Bill,  $x_1$ )

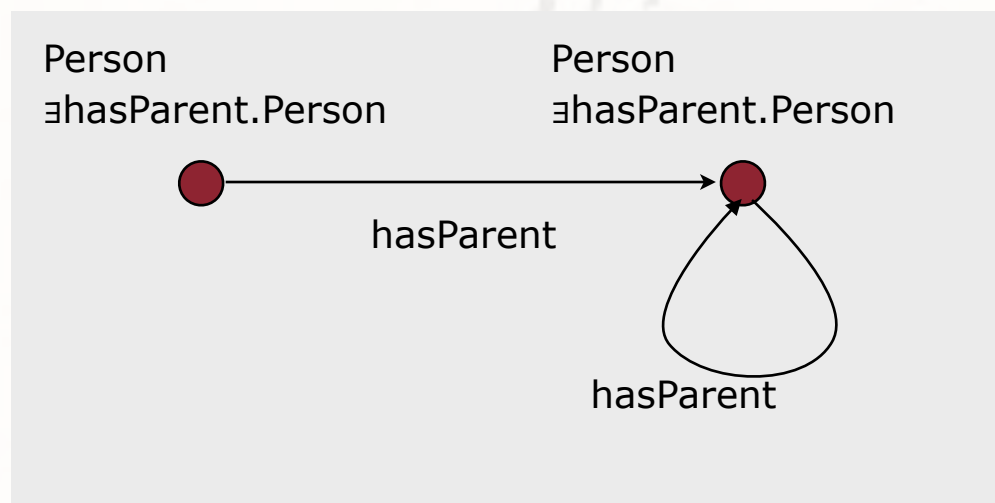
Person( $x_1$ )

$(\neg \text{Person} \sqcup \exists \text{hasParent}.\text{Person})(x_1)$

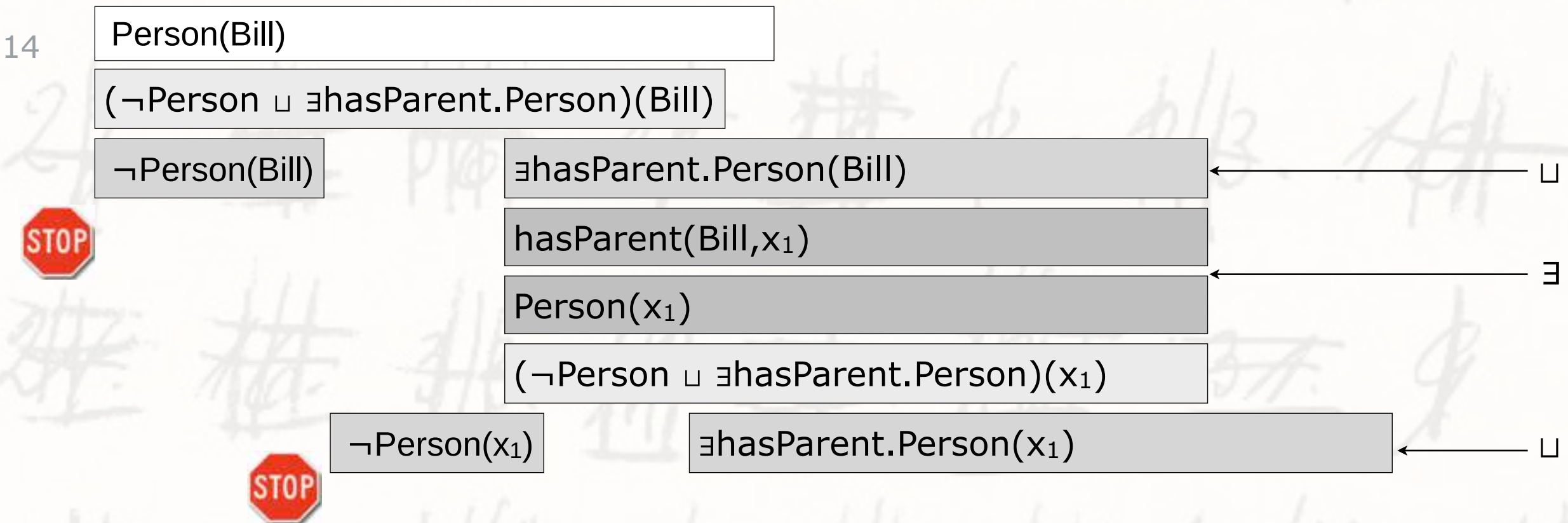
$\neg \text{Person}(x_1)$

$\exists \text{hasParent}.\text{Person}(x_1)$

termination



# Tableaux Algorithm (DL) with Blocking



$\sigma(\text{Individual}) = \{\text{Classes}\}$   
 $\sigma$  is **Class Extension** of an Individual

$\sigma(\text{Bill}) = \{\text{Person},$   
 $\neg \text{Person} \sqcup \exists \text{hasParent. Person},$   
 $\exists \text{hasParent. Person}\}$

$\sigma(x_1) = \{\text{Person},$   
 $\neg \text{Person} \sqcup \exists \text{hasParent. Person},$   
 $\exists \text{hasParent. Person}\}$

**$\sigma(x_1) \subseteq \sigma(\text{Bill})$ , therefore Bill blocks  $x_1$**



termination

# Tableaux Algorithm (DL) with Blocking

- <sup>15</sup> • The Selection of  $(\exists R.C)(a)$  in the tableaux path  $A$  is **blocked**, if there is already an individual  $b$  with  $\{C \mid C(a) \in A\} \subseteq \{C \mid C(b) \in A\}$ .
- Two possibilities of termination:

1. Closing the Tableaux.  
Knowledge Base is **unsatisfiable**.
2. All non blocked selections from the tableaux  
don't lead to an extension.  
Knowledge Base is **satisfiable**.



# Lecture 5: Knowledge Representations II

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